Free Will and Intensional Operators

Fabio Lampert and John William Waldrop
Forthcoming in the *Canadian Journal of Philosophy*

Abstract: Arguments challenging the existence of free will frequently share a common structure, relying on variants of a principle we call CLOSURE, according to which having no choice about a truth is preserved under entailment. We show that, under plausible assumptions, CLOSURE is valid if and only if the 'no choice' operator is *intensional*. By framing the debate in terms of the intensionality of this operator, this paper illuminates previously underappreciated constraints on defenses of CLOSURE-based arguments against the existence of free will.

1. Introduction

Philosophers have long noted challenges to the existence of free will arising from, say, the prospect of physical determinism or the existence of an all-knowing God. In principle, these challenges are not all interdependent. One might hold that free will is threatened by divine omniscience, for instance, while maintaining that a non-open future is no threat to free will (though presumably not *vice versa*). Some of these challenges, moreover, are more forceful than others.

These challenges can be formulated as compact and powerful arguments, and these arguments in turn have given rise to extensive philosophical discussion in recent decades (see the many articles in Fischer and Todd 2015; Campbell et al. 2023; among others). As we will see, several of the most prominent arguments challenging the existence of free will in the literature depend on a crucial and widely endorsed inference rule we call CLOSURE. So it is natural to ask: is this rule so much as *valid*?

In this article, we offer a partial answer to this question, which applies uniformly to the central arguments discussed here. First, we note that the validity of these arguments stands or falls with the validity of CLOSURE. We then show that CLOSURE is valid if and only if the 'no choice' sentential operator N is *intensional*, as defined below. In this way, a cluster of critical questions about free will spanning diverse areas of contemporary philosophy reduces to a single question of a kind familiar to logicians and formal metaphysicians: whether a central piece of philosophical ideology is intensional or hyperintensional.

In §2 we present four simple arguments corresponding to four challenges to free will and note that each instantiates the same inference rule involving the operator N, namely CLOSURE. In §3, we briefly motivate CLOSURE and explain its significance. In §4, we show that, under plausible background assumptions, CLOSURE is valid if and only if N is intensional. Thus, the validity of each of the four reconstructed arguments turns on whether N is an intensional operator. Importantly, we do not take a stand in this article on whether the N operator is in fact intensional; therefore we also remain neutral on the validity of the associated inference rule. Nevertheless, our arguments reveal nontrivial and previously underappreciated constraints on how several significant arguments in the free will literature can be defended. In light of these constraints, the final two sections offer an initial foray into the old debates made new. In §5, we examine arguments *in favor* of their validity by appealing to the intensionality of N; in §6, we consider arguments *against* their validity, focusing on counterexamples that suggest N is hyperintensional.

2. CLOSURE and Four Challenges to the Existence of Free Will

Challenges to the existence of free will have been a recurring theme in Western philosophy since classical times. Rather than surveying their historical development, we will focus instead on four simple arguments. As we will see, each of these arguments shares a common structure, relying on a familiar inference rule we call CLOSURE. To introduce the arguments, let us begin with a simple example, an everyday action that just about anyone could presumably freely perform. Suppose that you are about to raise your arm at some future time, t. Will you raise your arm freely at t? The following are four informal arguments for answering in the negative:[1]

(I) *The Argument from 'Prior Truths'*

(1) You have no choice about: it was true, long ago, that you would raise your arm at t.

(2) Necessarily, if it was true, long ago, that you would raise your arm at t, then you raise your arm at t.

---

(3) You have no choice about: raising your arm at t.

[1] In what follows, we proceed somewhat incautiously regarding the use-mention distinction. Accordingly, we will occasionally use unquoted sentence variables (e.g., 'p' and 'q') to refer to the corresponding sentences or propositions when the association is clear and doing so poses no risk of confusion.

(II) *The Argument from Divine Omniscience*

(1) You have no choice about: God believed, long ago, that you would raise your arm at t.

(2) Necessarily, if God believed, long ago, that you would raise your arm at t, then you raise your arm at t.

---

(3) You have no choice about: raising your arm at t.

(III) *The Argument from Determinism*

(1) You have no choice about: L and H (where L is a true proposition specifying the laws of nature, and H a true proposition specifying the intrinsic character of the world at some time in the remote past such that L and H jointly determined, long ago, that you would raise your arm at t).

(2) Necessarily, if L and H, then you raise your arm at t.

---

(3) You have no choice about: raising your arm at t.

(IV) *The Argument from Indeterminism*

(1) You have no choice about: DB and DB only if you raise your arm at t (where DB is the true proposition specifying your mental states prior to raising your arm at t—that is, the relevant desire + belief pair that causally explains your action).

(2) Necessarily, if DB and DB only if you raise your arm at t, then you raise your arm at t.

---

(3) You have no choice about: raising your arm at t.

These are simple versions of widely discussed arguments. Using more or less standard terminology, (I) is a reconstruction of the argument for *logical fatalism* (Taylor 1962; Haack 1974), (II) is a reconstruction of the argument for *theological fatalism* (Pike 1965; Fischer 2016), (III) is a version of the *Consequence Argument* for the incompatibility of free will and determinism (van Inwagen 1983; Huemer

2000), and (IV) is a version of the *Mind Argument* for the incompatibility of free will and indeterminism (van Inwagen 1983; Nelkin 2001).[2]

Those who are not free-will skeptics will be interested in these arguments mainly as furnishing what one may describe as *conditional* challenges to the existence of free will. Received in this spirit, the first argument shows that you have no choice about raising your arm *on the supposition* of a non-open future; the second does the same on the supposition of divine omniscience, etc.[3]

All four arguments above are schematic: in each case, the conclusion could involve *any* true proposition concerning a human action, and the premises stand in for the corresponding assumptions about future truths, divine knowledge, and so on. Assuming the arguments are valid, the upshot is that if their respective premises are true, then so are their conclusions; thus, no human ever acts freely.

In each argument, the first premise is typically supported by some version of the principle that, if p is a truth about the past, then it is not up to anyone whether p (at least not anymore). This claim, that the past is *fixed*, has been challenged in various ways (see Saunders 1966; Perry 2008; Merricks 2009, 2011; Dorr 2016; Lampert 2022, among others). Still, it remains a cornerstone of the case for the claim that free will is precluded by determinism, divine foreknowledge, and so on (see, e.g., Fischer and Todd 2013).[4]

---

[2] To be sure, there are various ways of formulating versions of these arguments, and a number of examples in the literature—including some of the works cited above—do not employ rules like CLOSURE. For instance, Ginet (1990), Fischer (1994), Mumford and Anjum (2014), and others, present challenges to the existence of free will from determinism without using CLOSURE. Still, as discussed below, many arguments *do* rely on CLOSURE and many philosophers take CLOSURE-based arguments to pose formidable problems for free will. Fischer (2016: ch. 1), Campbell (2016), and Campbell and Lota (2023) classify and discuss different types of arguments challenging free will, some of which are not based on CLOSURE.

[3] Such conditional challenges to free will are related to familiar theses to the effect that free will is *incompatible* with, for example, determinism, but it is important not to conflate the former with the latter, nor to conflate arguments for the former with arguments for the latter. Even if the above arguments are all valid—and a great many incompatibilists of different kinds, as discussed below, would accept that they are valid—there remain outstanding issues if they are taken to be arguments for related incompatibility theses. To take one example, the rules of inference on which versions of the Consequence Argument (see below) are usually based could be valid, but as an argument for the incompatibility—which is a modal thesis—of determinism and free will the Consequence Argument would still face familiar and outstanding difficulties, such as the *no-past objection*, as developed in Campbell (2007) (see Campbell 2016: 157–158, for discussion). We should note, however, that since most defenders of arguments such as the above *are*, in fact, defenders of related incompatibilist theses, we will occasionally refer to defenders of such arguments as 'incompatibilists'—that is, despite the fact that the arguments above do not need to be taken as arguments for modal, incompatibilist claims such as that determinism *necessarily* precludes free will.

[4] Each argument also admits other, less prominent challenges to the first premise. In the third argument, for example, one might follow Lewis (1981) in arguing that the laws of nature are, in some sense, up to us. In all four arguments, one might deny the prior *existence* of certain truths about future human actions, so that it would not have been true *in the past*, or *long ago*, that you would raise your arm at t. For discussion related to the former response, see Dorr (2016); for a

The second premise in each argument is relatively uncontroversial. In (I), it can be motivated by the general idea that prior truths about later times settle how things must be at those later times. In (II), it reflects the widely accepted principle of divine infallibility, according to which God could not hold false beliefs. In (III), the premise follows from determinism: for any truth p, the conjunction of L and H entails p. In (IV), the second premise corresponds to a theorem of standard modal logic: $\Box((p \wedge (p \rightarrow q)) \rightarrow q)$.

Furthermore, the arguments in question all share the same "shape". Where p is any truth:

(1) S has no choice about: p.

(2) Necessarily, if p then q.

---

(3) S has no choice about: q.

More schematically, let 'Np' abbreviate 'p and S does not have, never had, and never will have a choice about whether p'—or, simply, 'p and S has no choice about whether p'. Then, arguments (I)–(IV) each instantiate an inference rule that aims to derive (3) from premises (1) and (2), namely:

CLOSURE: $Np, \Box(p \rightarrow q) \vdash Nq$

If CLOSURE is valid, then each of the four arguments above is likewise valid, and the central question becomes whether their first premises are true. If, on the other hand, CLOSURE is invalid, then the arguments are unsound. It is therefore crucial to assess the plausibility of CLOSURE itself.[5]

survey of the latter, see Lampert (2025: 5–7).

[5] A brief remark on our use of the term 'valid': while validity could be understood in the strict logical sense—as captured by the logician's formula that logical validity is truth preservation in virtue of logical form—that is not essential for our purposes here. There is considerable debate over what logical validity amounts to, and many accounts underdetermine whether a rule like CLOSURE (which involves modal operators such as $\Box$, not to mention N) could count as logically valid. In what follows, our concern is simply with whether rules like CLOSURE are *materially truth-preserving*—that is, whether there could be a uniformly interpreted *material counterexample* to the principle: a case in which the premises are true and the conclusion false, under a single, uniform interpretation. The rationale for focusing on material truth preservation is straightforward: we are evaluating CLOSURE in the context of standard arguments in the free will literature, where what ultimately matters is whether such rules guarantee true conclusions given true premises, irrespective of whether the guarantee is underwritten by those rules having some special status we call 'logical'. Accordingly, we use the term 'valid' somewhat loosely—though in a way that is both germaine to the topic at hand and consistent with common usage in the relevant literature.

3. Whence CLOSURE?

To better understand CLOSURE in the context of the free will literature, it is helpful to briefly summarize some relevant recent history. CLOSURE was first formulated in connection with van Inwagen's Consequence Argument for the incompatibility of free will and determinism. In his original presentation, van Inwagen introduced the 'no choice' operator N, where 'Np' abbreviated 'p, and no one has, or ever had, any choice about whether p'. To explicate this operator, van Inwagen invoked the notion of *rendering a proposition false*: one has no choice about a truth if and only if one is not able to render it false.[6] In this sense, if I ordered coffee after lunch, though I could have ordered tea or nothing at all, then I had a choice about the fact that I ordered coffee, since there was something I could have done such that, had I done it, the proposition *that I ordered coffee* would have been false.

Van Inwagen then proposed the following inference rules involving the operator N:

ALPHA: $\Box p \vdash Np$

BETA: $Np, N(p \to q) \vdash Nq$

According to ALPHA, no one has a choice about necessary truths. According to BETA, if no one has a choice about a truth p and no one has a choice about the conditional $p \to q$, then no one has a choice about q. In other words, choicelessness distributes across material conditionals that no one has a choice about.

We can now present van Inwagen's original Consequence Argument. Let p be any truth about a human action—for instance, the fact that you will (or did) raise your arm. It is a definitional consequence of determinism that the conjunction of the laws of nature L and a complete description of the remote past H together necessitate every truth (see premise (1) of argument (III)). Thus:

1.	$\Box((L \wedge H) \to p)$	Premise, from Determinism
2.	NL	Premise, Fixity of the Laws of Nature
3.	NH	Premise, Fixity of the Past
4.	$\Box(L \to (H \to p))$	1, by Modal Exportation
5.	$N(L \to (H \to p))$	4, by ALPHA
6.	$N(H \to p)$	2 and 5, by BETA
7.	Np	3 and 6, by BETA

Since p is arbitrary, given determinism, it only matters whether p is true. But p is also about human action. Since the same line of reasoning applies to any truth

[6]See van Inwagen (1983: 66–67). For discussion on the notion of *rendering a proposition false*, see Lewis (1981), Huemer (2000), Schnieder (2004), and van Inwagen (2017). Van Inwagen also proposed different readings of *N*, as in van Inwagen (2015: 19).

whatsoever, *a fortiori* it applies to any truth concerning human actions. Thus, if determinism is true, no one has a choice about any human action. Therefore, no one has free will.

The derivation itself relies on two extralogical inference rules, namely ALPHA and BETA. But are these rules valid? What can we say in their favor?

There are no standard arguments in defense of ALPHA. Its validity is typically taken to be self-evident and has only rarely been questioned (see, for instance, §6).[7] Van Inwagen claims that no one could or would dispute the validity of ALPHA, and asserts that it is obviously valid (see van Inwagen 1989: 405; 1990: 283; 2000: 2). Vihvelin (1988: 230) remarks that ALPHA "seems uncontroversial", while Finch and Warfield (1998: 517) describe it as "surely unobjectionable". McKay and Johnson (1996: 115) go so far as to call it "impeccable", adding that it "needs no defense".

So much for ALPHA; what about BETA? Van Inwagen (1983: 96) himself concedes that it is "the most difficult of the premisses (...) to defend". As it turns out, BETA is now widely regarded as invalid. The most familiar counterexamples to BETA are also counterexamples to another inference rule, namely AGGLOMERATION:

AGGLOMERATION: $Np, Nq \vdash N(p \wedge q)$

AGGLOMERATION is derivable from ALPHA and BETA.[8] Therefore, a counterexample to AGGLOMERATION would be a counterexample to ALPHA and BETA taken together, thereby undermining van Inwagen's formulation of the Consequence Argument. Proposed counterexamples to AGGLOMERATION, canonically formulated by McKay and Johnson (1996), are widely known. Pruss (2013: 431) considers their counterexamples to be "conclusive", and van Inwagen himself (2000) claims that they undermine his original rule BETA. Since ALPHA is widely accepted and AGGLOMERATION is derivable from ALPHA and BETA, there is an entrenched consensus that BETA is therefore invalid.[9]

In response, some defenders of the Consequence Argument have proposed replacing BETA with a different principle, namely, what we above called CLOSURE.[10]

[7] See van Inwagen (1983: 96).

[8] Assume *Np* and *Nq*. Since $p \rightarrow (q \rightarrow (p \wedge q))$ is a tautology, it can be necessitated; therefore, $\Box(p \rightarrow (q \rightarrow (p \wedge q)))$ and $N(p \rightarrow (q \rightarrow (p \wedge q)))$ are deducible by necessitation and ALPHA, respectively. Now, $N(q \rightarrow (p \wedge q))$ follows from the latter together with the first assumption by BETA, and, finally, $N(p \wedge q)$ follows from it together with the second assumption by BETA.

[9] Potential counterexamples formulated directly against BETA can be found in Widerker (1987) and Huemer (2000). Whether BETA and AGGLOMERATION are valid depends, however, on more subtle questions about counterfactual conditionals, which we have no space to discuss here. See Merlussi (2022) for more details.

[10] CLOSURE is sometimes referred to as BETA-2 in the literature. The strategy of replacing BETA with CLOSURE was first suggested by Widerker (1987), and later defended by Finch and Warfield (1998).

Replacing BETA with CLOSURE allows for a streamlined and more elegant formulation of the Consequence Argument:

1.	$N(L \wedge H)$	Premise, Fixity of the Laws of Nature and the Past
2.	$\Box((L \wedge H) \rightarrow p)$	Premise, from Determinism
3.	Np	1 and 2, by CLOSURE

The validity of this argument clearly hinges on the validity of CLOSURE alone, as the argument is simply an instance of it. This argument may also be regarded as a regimentation of (III) above.[11]

CLOSURE is now a standard feature of arguments challenging the existence of free will. For instance, variants of arguments (I) and (II) appealing to CLOSURE have been presented and discussed by Finch and Warfield (1999), Rea (2006), Finch and Rea (2008), Fischer and Todd (2015), Merricks (2009, 2011), Todd (2023), Lampert (2025), among others. Nelkin (2001) develops a prominent version of (IV) employing CLOSURE as well. More generally, some of the most prominent contemporary arguments for various forms of *incompatibilism* all depend straightforwardly on CLOSURE.

But the significance of CLOSURE is hardly limited to those arguments alone. For example, the validity of van Inwagen's original Consequence Argument also requires the validity of CLOSURE. Recall that the original argument invokes the rules ALPHA and BETA. From these, however, one can straightforwardly derive CLOSURE:

1.	Np	Premise
2.	$\Box(p \rightarrow q)$	Premise
3.	$N(p \rightarrow q)$	2, by ALPHA
4.	Nq	1 and 3, by BETA

Thus, defenders of the original Consequence Argument *presuppose*, in a sense, the validity of CLOSURE, for any counterexample to CLOSURE would undermine ALPHA and BETA, and with it van Inwagen's argument.[12]

[11] The conclusion of this argument, as well as of van Inwagen's original Consequence Argument, is simply Np. Whether such arguments and inference rules warrant further inferences to a modal conclusion, that is, to a formal regimentation of the claim that determinism necessarily precludes free will (i.e. the thesis typically defended by incompatibilists), is contentious. See footnote 3.

[12] In response to McKay and Johnson, van Inwagen proposed a revised reading of the N operator, which, he claims, preserves the plausibility of the original rules ALPHA and BETA, even in the face of their counterexamples. The idea is that those counterexamples would show only that AGGLOMERATION is problematic under stronger interpretations of N, such as the original one. It is nevertheless important to emphasize that the foregoing point holds independently of one's preferred interpretation of the N operator. Thus, even if a revised reading of N removes the perceived need to adopt CLOSURE in order to avoid the difficulties with AGGLOMERATION, any interpretation, including van Inwagen's, that aims to preserve the original rules ALPHA and BETA must also preserve CLOSURE.

So, what can be said in favor of CLOSURE?

With a few exceptions (which we will discuss in due course), it is only the apparent obviousness of CLOSURE that has counted in its favor—and that almost conclusively. For this reason, partisans of CLOSURE rarely offer arguments in its defense. Finch and Warfield (1998: 522) remark:

> The inference principle we employ is clearly less vulnerable to criticism than van Inwagen's [BETA]. We claim only that one has no choice about the logical consequences of those truths one has no choice about. Because no one has a choice about the past and laws of nature, and because the future is, given determinism, a logical consequence of the past and laws, [CLOSURE] licenses the inference from determinism to the conclusion that there is no free will. [CLOSURE] also avoids the McKay and Johnson counterexample by blocking the derivation of the invalid [AGGLOMERATION] principle ...

Elsewhere, they maintain in the same spirit that CLOSURE is "clearly valid" (Finch and Warfield 1998: 525). Similarly, Kane (2007: 11), in the absence of a compelling defense of CLOSURE, appeals to intuitive plausibility and illustrative examples. On similar grounds, he claims in an earlier work that CLOSURE is "as plausible" as ALPHA (Kane 2005: 26). It is also worth noting that principles closely resembling CLOSURE appear in related contexts, particularly in debates about the compatibility of determinism and moral responsibility. Like CLOSURE, these principles are rarely defended through rigorous argumentation, yet they are frequently regarded as intuitively compelling.[13]

There is, then, widespread acceptance of CLOSURE (and closely related principles) among philosophers, which is motivated largely by appeals to intuition rather than by rigorous (let alone formal) argument. (In the following section we note a few exceptions.) Our main focus, however, is a simple and powerful argument for CLOSURE. Reflecting on this argument leads us to our central contention: that the key issue in the debate over CLOSURE is whether the *N* operator is intensional or hyperintensional.

[13] Warfield's (1996) reformulation of van Inwagen's *Direct Argument* for the incompatibility of moral responsibility and determinism employs a responsibility-theoretic analogue of CLOSURE, called BETA □. Warfield writes: "I have no argument for the validity of Rule BETA □. I can think of no example which demonstrates that it is not valid, nor have I found anyone who can produce such an example" (1996: 216). Relatedly, we note that van Inwagen's original formulation of the Direct Argument (1983: 182-188) appeals to principles that are in effect corresponding responsibility-theoretic analogues of ALPHA and BETA. In general, what we say about the *N* operator and its proposed logic applies, *mutatis mutandis*, to close responsibility-theoretic analogues in the literature on the Direct Argument. Interesting connections between discussions of the Direct Argument and our present subject matter are plentiful, but since many of these are rather obvious, we will rarely mention them in what follows.

4. The Intensional Argument

Scattered attempts to vindicate CLOSURE and related principles are available, though these are almost exclusively confined to the literature on the Consequence Argument. Notably, informal arguments in support of CLOSURE can be found in Carlson (2000) and Huemer (2000), while formal derivations are presented in Pruss (2013). Despite their clear merits, existing strategies for defending CLOSURE are explicitly narrow in scope: they proceed by first identifying plausible, but nevertheless highly specific, interpretations of the N operator, and then arguing that *those* interpretations validate CLOSURE.

To clarify, note that we do not expect the sentence 'p and no one can render p false' to wear its meaning on its sleeve in ordinary discourse. The modal verb 'can' serves a wide variety of roles, and even within the class of so-called "agentive" or "ability" modals, its truth-conditions in a given context are highly context-sensitive. That is to say nothing of the expression 'render false', which, in this context, is surely a philosopher's invention.

Now, one cannot sensibly argue that CLOSURE is a truth-preserving inference rule without addressing the truth-conditional interpretation of the sole non-logical constant occurring therein, namely the operator N. Accordingly, defenders of CLOSURE typically proceed by identifying a single interpretation of N, and then arguing that CLOSURE, so interpreted, is valid. One may attempt to isolate a suitable reading either by appealing to an "intended interpretation" of the operator N, or by stipulating a particular technical reading. As a result, the free will literature has witnessed a proliferation of proposed interpretations of N.[14]

But this narrow strategy is hardly the only way to defend CLOSURE, since one can profitably address the truth-conditional interpretation of N without committing to any highly specific reading. For obvious reasons, more general strategies for vindicating CLOSURE are likely to be more promising, if feasible. We believe that one such broadly applicable strategy is indeed feasible.

Very roughly, instead of focusing on the meaning of 'no choice', or on any single reading of the N operator, we propose investigating the behavior of N within the framework of a background normal modal logic, under *any* interpretation that satisfies minimal formal constraints. This amounts to a somewhat superficial investigation of the bimodal logic of the operators $\Box$ and N. One advantage of this approach is that it allows us to set aside narrower questions about how N should be understood—questions that are difficult at best and merely verbal at worst. Any results of this investigation will thus be extremely general.[15]

[14] In addition to the readings discussed in Huemer (2000), Carlson (2000), and Pruss (2013), and in works by van Inwagen (1983, 2000, 2015, 2017), recent, more formal discussions of related topics can be found in Schnieder (2004), Lampert and Merlussi (2021a), Merlussi (2022), de Rizzo (2022), and Waldrop (2023).

[15] Since previous defenses of CLOSURE have typically relied on specific interpretations of N, and since our investigation is highly general, the arguments presented here promise to subsume and

Even on the most cursory consideration of the issues, two minimal requirements on the logic of the N operator readily suggest themselves. First, N is *factive*:

FACTIVITY: $Np \vdash p$

Second, N is *closed under conjunction elimination*:

CONJUNCTION: $N(p \wedge q) \vdash Np$

FACTIVITY is straightforwardly borne out by every proposed reading of the N operator to date, and for this reason we treat it as part of the common ground. After all, N is definitionally only applied to *truths* that no one has any choice about.

CONJUNCTION is almost as compelling as FACTIVITY. Nevertheless, since we cannot plausibly claim that CONJUNCTION holds by definition alone, it is worth offering some straightforward, if nontrivial, remarks in its support.

First, under any intuitive characterization of the N operator, rejecting CONJUNCTION leads to paradoxical results. Suppose there is a counterexample to CONJUNCTION. Then: (a) someone has no choice about the conjunction $p \wedge q$; (b) by FACTIVITY, $p \wedge q$ is true; and yet (c) someone does have a choice about one of the conjuncts, say, p. But, following van Inwagen (1983), having a choice about a truth is very close to simply being able to "render it false" (however this is understood). Taken together, we then get the paradoxical conclusion that someone could render p false without having any choice about whether $p \wedge q$ is true—that is, without being able to render $p \wedge q$ false.

There are nevertheless stronger reasons for favoring CONJUNCTION. If other principles proposed by incompatibilists for governing the N operator hold, then so does CONJUNCTION. The principles in question are the standard inference rules appealed to in different versions of the Consequence Argument. In particular, CONJUNCTION can be proved from ALPHA and BETA, regardless of how the N operator is understood.[16] If we drop the pairing of ALPHA and BETA in favor of CLOSURE alone, we obtain the same conclusion.[17]

Finally, a more modest argument for CONJUNCTION assumes only that N is Boolean. That is, suppose N is closed under the consequence relation of the classical propositional calculus: given Np and some classical consequence q of p, we infer Nq. Since any conjunction has each of its conjuncts as tautological consequences, CONJUNCTION follows immediately.

---

generalize those earlier defenses. A detailed examination of this point, however, lies beyond the scope of the present paper.

[16]Assume $N(p \wedge q)$. As $(p \wedge q) \rightarrow p$ is a tautology, $\Box((p \wedge q) \rightarrow p)$ is derivable from it by necessitation, so $N((p \wedge q) \rightarrow p)$ is in turn derivable by ALPHA. From this and the assumption, Np follows by BETA.

[17]Assume $N(p \wedge q)$. As before, $\Box((p \wedge q) \rightarrow p)$ is a necessitated tautology, and therefore Np follows from it and the assumption by CLOSURE.

In this way, CONJUNCTION can be seen as a logical common core underlying the principles previously recommended for governing the *N* operator. However, we are *not* arguing for CONJUNCTION on the *assumption* that the more substantive rules governing *N* are truth-preserving; doing so would be dialectically backwards. Rather, we proceed on the assumption that defenders of CLOSURE have *some* understanding of what 'no choice' means in their mouths. Thus even if their endorsement of principles like CLOSURE does not guarantee that such rules *are* truth-preserving, it still provides evidence that violations of principles like CONJUNCTION involve a change of subject matter.

Those are our two proposed minimal requirements on the logic of *N*. How might we more adventurously move beyond them? What further, general considerations about the *N* operator can be brought to bear on the question of whether CLOSURE is valid? As it turns out, venturing just a little farther, we can say quite definitively what it takes for CLOSURE to be truth-preserving. There is a simple, straightforward proof of CLOSURE that follows from our minimal logical assumptions together with one further condition,namely, that *N* is an *intensional operator*.

To clarify, contemporary philosophers use the word 'intensional' and its cognates in diverse ways. Adhering to a somewhat dated usage, in some quarters 'intensional' is used interchangeably with 'non-extensional'. According to this older usage, any sentential context in which *material* equivalents cannot be substituted truth-preservingly for one another counts as an intensional context. This, however, is not the sense of 'intensional' we have in mind here.

Whereas the older usage emphasizes the contrast between the extensional and the non-extensional, more recent usage emphasizes the contrast between the *hyperintensional* and the *non-hyperintensional*. Roughly speaking, a hyperintensional sentential context is one in which necessarily equivalent expressions are not guaranteed to be truth-preservingly substitutable, or substitutable *salva veritate*. A paradigmatic hyperintensional context is an open attitude ascription. Consider the sentential context 'Peter believes that ...'. Philosophical lore holds that from the sentence 'Peter believes that Superman flies', one cannot truth-preservingly deduce 'Peter believes that Clark Kent flies', even though the two embedded complement sentences are necessarily equivalent—that is, the propositions expressed by the two sentences are true in exactly the same possible worlds. Such sentential contexts are regarded as paradigmatically hyperintensional. Thus, rather than taking *intensional* as contrasting primarily with *extensional*, more recent usage contrasts *intensional* with *hyperintensional*. This is the usage we adopt in what follows.

In contrast to hyperintensional sentential operators, *intensional* operators thus permit the substitution of necessary equivalents into operand position *salva veritate*. A paradigmatic intensional operator is the *necessity* operator. Necessity is intensional, for if p and q are necessarily equivalent, $\Box p$ and $\Box q$ have the same truth value. We can emphasize the intensionality of $\Box$ by noting that, whereas philosophical lore does *not* want the following conditional to come out true:

Peter believes that Superman flies only if Peter believes that Clark Kent flies,

it *does* take the following to be true:

Superman necessarily flies only if Clark Kent necessarily flies.

Thus, the lore dictates that belief contexts are *hyper*intensional, whereas the necessity operator is intensional.

So, how might one regiment the thesis we wish to consider, that is, the thesis that N is an intensional operator? For our purposes, it will suffice to explicate this thesis by means of the following inference rule:[18]

INTENSIONALITY: $Np, \Box(p \leftrightarrow q) \vdash Nq$

Hereafter, when we say that N is an intensional operator, we stipulatively mean that INTENSIONALITY holds.

As advertised, we are now in a position to state what it takes for CLOSURE to be truth-preserving. Given our minimal assumptions, along with the additional assumption that N is an intensional operator, a short proof of CLOSURE is readily available:

1.	Np	Premise
2.	$\Box(p \rightarrow q)$	Premise
3.	$\Box(p \leftrightarrow (p \wedge q))$	2, by Modal Logic
4.	$N(p \wedge q)$	1 and 3, by INTENSIONALITY
5.	Nq	4, by CONJUNCTION

Call this argument for CLOSURE the *Intensional Argument*.

At the risk of repetition, we note again that the Intensional Argument proceeds independently of any particular reading of N, thereby distinguishing it from extant defenses of CLOSURE. In this sense, the argument is highly general: however N is defined, the Intensional Argument shows that CLOSURE is valid so long as N is intensional. Note also that if CLOSURE is valid, then so is INTENSIONALITY; if N is closed under entailment, it is *a fortiori* closed under necessary equivalence. Thus, against our background assumptions, the validity of INTENSIONALITY is

[18]We can generalize INTENSIONALITY to better align with a more precise characterization of the intensionality of N: where ϕ is any non-hyperintensional sentential context,

INTENSIONALITY*: $N\phi(...p...), \Box(p \leftrightarrow q) \vdash N\phi(...q...)$

Simply put, INTENSIONALITY* codifies the logical assumption that necessarily equivalent expressions are intersubstitutable within the scope of N—provided, at least, that the scope of N does not itself contain any hyperintensional contexts.

both necessary and sufficient for the validity of CLOSURE. This, then, is what it takes for CLOSURE to be truth-preserving: N must be an intensional operator.[19]

Thus, we arrive at the central contention of this paper: the status of CLOSURE—and with it, all four of the arguments identified earlier—turns on whether N is an intensional operator. If we reframe disputes about these arguments in terms of the narrower and more precise question of whether N is intensional, there is real hope for more progress on some of the grand old questions surrounding free will. After all, debates about intensionality and hyperintensionality are well-trodden territory for contemporary logicians, philosophers of language, and formally inclined metaphysicians. In the next two sections, we undertake a preliminary exploration of these questions.

5. *N* as an Intensional Operator

As the previous section argues, CLOSURE is valid if N is an intensional operator. But if CLOSURE is valid, so are the four arguments discussed above that challenge the existence of free will. Thus, if we can show that N is an intensional operator, we will thereby have an answer to the open question whether the four arguments are valid—namely, that they *are* valid. So, what can be said in favor of the thesis that N is intensional?

Thus far, the literature on free will has largely overlooked this question. However, some observations drawing on the existing literature *do* suggest that N should be treated as an intensional operator. For one thing, an argument for the intensionality of N was already available to van Inwagen when he originally formulated the Consequence Argument. In that setting, INTENSIONALITY is implicit in two of his operational assumptions. First, to have a choice about p is to stand in some special relation R to the proposition that p. Second, propositions are identified with sets of possible worlds: that is, the proposition that p is the set of worlds in which p is true (see van Inwagen 1983: 58–59).

Given these assumptions, it follows that N is intensional, and, by the argument of the previous section, that CLOSURE is valid. For suppose that Np is true and that $\Box(p \leftrightarrow q)$ holds, so that p and q are true in exactly the same possible worlds. Given the possible-worlds theory of propositions, the proposition that p just is the proposition that q. Given that Np, no one has a choice about whether p, and so no

[19]The Intensional Argument also highlights the fact that if N is an intensional operator, then CLOSURE is valid if and only if ALPHA is valid. This follows immediately from the fact that ALPHA is derivable from CLOSURE. Strictly speaking, ALPHA is derivable from CLOSURE given the trifling assumption that Nq holds for some q—that is, there is at least one truth about which no one has any choice. The proof is straightforward. Assume $\Box p$ and Nq. As p is a necessary truth, it is strictly implied by anything, and so $\Box(q \to p)$ holds. But from this and the second assumption, Np follows by CLOSURE, as desired. Given INTENSIONALITY, then, any counterexample to CLOSURE would also be a counterexample to ALPHA—that is, to the principle that no one has a choice about necessities.

one stands in relation *R* to the proposition that p. But then, given the aforementioned identity, it follows that no one stands in relation *R* to the proposition that q; hence, Nq is true.

Note that this argument does not really require endorsing a possible-worlds theory of propositions—something van Inwagen himself was unwilling to do. In the passage referred to above, van Inwagen suggests that such a theory is *not* serviceable as a general account of propositions. Nevertheless, he maintains that it *is* serviceable for clarifying the key technical notions at issue in the Consequence Argument. Sensibly enough, then, we need only say—more modestly—that, for van Inwagen, the most distinctive and bold feature of such a theory, namely, that it individuates propositions by necessary equivalence, generates the right predictions about the interaction of the standard modal operators on the one hand and the *N* operator on the other. This modest retreat leaves the above argument essentially unaltered.

A further remark by van Inwagen supports treating the *N* operator as intensional. At one point in *An Essay on Free Will*, van Inwagen suggests the possibility of giving a basic possible-worlds interpretation of the *N* operator:

> (...) one might employ the methods of formal semantics. In the present case, since 'N' is a modal operator, the methods of *possible-world* semantics might seem promising. Here is a sketch of how we might apply these methods (...) We first delimit a certain set W of worlds and say that N*p* is true just in the case that *p* is true in all these worlds. This would amount to a semantical definition of 'N'. (1983: 96–97)

We could generalize this proposal by positing compositional possible-worlds truth-conditions for sentences formed with the operator *N*. This is a highly plausible proposal, since it effectively treats 'no one has a choice about whether *p*' in a manner analogous to standard treatments of agentive modals in formal semantics, which draw heavily on the influential work of Kratzer (1977).

As it happens, this proposal also straightforwardly validates INTENSIONALITY. To see why, suppose that *N* is defined in the way van Inwagen describes. Then there exists some set *S* of possible worlds such that Np is true if and only if p is true in all worlds in *S*. Now suppose that Np is true. Then p is true throughout *S*. If $\Box(p \leftrightarrow q)$ is also true, then p and q are true in exactly the same possible worlds. It follows that q must also be true in all worlds in *S*, and hence Nq is true. Thus, *N* satisfies INTENSIONALITY.[20]

[20] A related but somewhat weaker point concerns the principles van Inwagen invokes elsewhere in connection with the *N* operator—namely the inference rules ALPHA and BETA. As noted above, if the necessity operator $\Box$ obeys a normal modal logic, then these two rules suffice to determine a normal modal logic for the *N* operator as well. Such logics can be modeled using Kripke semantics, which involve accessibility relations over a set of possible worlds. But any model-theoretic interpretation of this sort is, in the relevant sense, intensional, thereby supporting INTENSIONALITY as a natural consequence of the logic determined by ALPHA and BETA.

One further thing these considerations bring out is that any characterization of *N*—or informal understanding of 'no choice'—that appeals solely to intensional notions will itself support INTENSIONALITY. Paradigmatic intensional notions of this sort include those involving counterfactual dependence or causal relations. This observation connects our present discussion to the existing literature in two important respects. First, in the literature on the Consequence Argument several readings of *N* have in fact been proposed that rely exclusively on intensional resources. Given our minimal formal assumptions, the Intensional Argument shows that such interpretations will validate CLOSURE, and thereby all four arguments discussed above. Second, the Intensional Argument offers a unifying framework that generalizes certain arguments already present in the literature for the validity of CLOSURE. Indeed, the assumptions behind the Intensional Argument are sufficiently modest that plausibly the best existing defenses of CLOSURE are special cases of the Intensional Argument.

This very brief discussion highlights underexplored routes to the conclusion that the four arguments challenging free will from §2 are all valid, namely, through the metaphysics of propositions and the semantics of ability modals. Moreover, there are further avenues for exploration that we have yet to consider: seasoned metaphysicians, for instance, may find promising approaches by appealing to well-known general theses about supervenience, intrinsicality, or grounding, all of which can be connected to necessity and possibility in various ways. If it can be argued convincingly that *N* is insensitive to putative differences between necessary equivalents, the implications for the free will literature would be far-reaching.

That said, the Intensional Argument also motivates a distinct line of inquiry for those who wish to argue that one or more of the arguments challenging free will we focus on are *invalid*. It is to this investigation that we now turn.

6. *N* as a Hyperintensional Operator

What the Intensional Argument clarifies is that if *N* is an intensional operator, then CLOSURE, and with it the four arguments discussed in §2, are valid. Accordingly, any counterexample to CLOSURE must also be a counterexample to INTENSIONALITY; that is, it must involve *N* being a *hyperintensional* operator. Parallel to the last section, this insight motivates us to propose some general considerations against the validity of CLOSURE drawing on discussions beyond the free will literature.

Let us outline one such consideration here. The following three theses enjoy some popularity in contemporary philosophy:

(A) All intentional action is acting on know-how (Anscombe 1957).

(B) All know-how is propositional knowledge (Williamson and Stanley 2001).

(C) Propositional knowledge is hyperintensional.

Together, these three theses make it plausible that N is hyperintensional. Given (B) and (C), it is plausible that someone might, on occasion, know how to render p false despite not knowing how to render q false, even when p and q are necessarily equivalent. Consequently, given (A), it is also plausible that one might be able to perform the intentional action of rendering p false without fulfilling a necessary requirement on being able to render q false. Thus, if the intended sense of N is limited to the ability to perform *intentional* actions, theses (A)–(C) suggest that N is hyperintensional. If, moreover, such possible cases show that N is hyperintensional, they serve as *counterexamples* to INTENSIONALITY and, therefore, make it plausible that CLOSURE is invalid.

It is therefore not only defenders of CLOSURE who can advance new arguments in light of the Intensional Argument, as the latter also provides systematic guidance to detractors of standard incompatibilist arguments. To emphasize this point, we will next highlight a family of possible counterexamples to CLOSURE and show how they conform to the pattern suggested by the Intensional Argument—namely, that these are cases where one would be inclined to regard N as hyperintensional.

To begin, suppose someone has a choice about a necessary truth. It is not hard to see why such cases would challenge CLOSURE. Treated abstractly, suppose p is a truth that no one has a choice about, and that q is a necessary truth that someone does have a choice about. Since q is necessary, $\Box(p \to q)$ is true. So, both Np and $\Box(p \to q)$ are true. Nonetheless, because someone has a choice about q, Nq is false. We thus have a counterexample to CLOSURE.

Now, for our purposes, all we require are cases where *someone* has a choice about *some* necessary truth. Moreover, it does not matter for our purposes whether these cases are rare or marginal. With that said, let us begin by considering the following principle:

(POSS) If S is able to A, then it is metaphysically possible for S to A.

Spencer (2017) presents several tempting counterexamples to (POSS), and these can easily be extended to counterexamples to CLOSURE. Consider, once more, the conjunction of L and H, as specified in (III). Suppose that an agent, G, is able to know $L \wedge H$ but does not in fact come to know it—for instance, perhaps G was absent the day $L \wedge H$ was taught in school. Since knowledge is factive, if G were to know $L \wedge H$, then $L \wedge H$ would be true. Given the deterministic nature of L, any possible world in which $L \wedge H$ holds must be a duplicate of the actual world. But, by assumption, G does not know $L \wedge H$ in the actual world. It follows that G does not know $L \wedge H$ in any world where $L \wedge H$ holds. Therefore, it is impossible for G to know $L \wedge H$. Nonetheless, by hypothesis, G is *able* to know $L \wedge H$. Thus, G is able to do the impossible.[21] If Spencer is right, then G has a choice, in the relevant sense, about whether she does not know $L \wedge H$, even though it is *necessary* that she does not know $L \wedge H$. This presents a case in which someone has a choice

[21] See Spencer (2017: 468–469).

about a necessary truth; if a case like this is possible, there are counterexamples to CLOSURE.

Now, even if cases like this are *bona fide* counterexamples to CLOSURE, we can only appreciate them by assuming from the outset that an agent inhabiting a deterministic world might possess unexercised—indeed, *necessarily* unexercised—abilities. But this is precisely what the standard incompatibilist denies: according to the incompatibilist about determinism and free will, agents in a deterministic world are only able to do what they in fact do. For this reason, such cases are dialectically inert when evaluating incompatibilist arguments.[22]

A less recherché variation on this theme comes from Lampert and Merlussi (2021a):

> (...) let ϕ be the statement, say, that Nina raises her hand. Then someone has a choice about whether ϕ is true, namely Nina herself. Moreover, if Nina actually exists, she has a choice about whether ϕ is true *at the actual world*. But from this fact it is reasonable to conclude that she also has a choice about whether the actualized statement $@\phi$ is true at the actual world, even though $@\phi$ and ϕ have different modal profiles: the former is necessary if true, whereas the latter is only contingent. (2021a: 452)[23]

The case as described employs the standard rigidifying 'actually' operator, @.[24] Though facts about how contingent matters stand are contingent, facts about how contingent matters *actually* stand are not. This idea makes intuitive sense if we reason our way through it in terms of possible worlds. Let 'α' designate the actual world. Truths about what things are like at α are noncontingent: facts about what is true may depend on which world is actual, but facts about what is true *at a particular possible world* do not depend on which world is actual. So, suppose $@p$ is true. Then p is true at α. But at any other world (accessible from α) it will be true that p is true at α, and so it will be true that $@p$; therefore, $@p$ is necessarily true. Thus Nina actually raises her hand only if she *necessarily* actually raises her hand. Now suppose Nina has a choice about whether she raises her hand. Then, plausibly enough, she has a choice about whether she *actually* raises her hand. But then Nina

[22] We note, however, that Spencer suggests an alternative route to necessarily unexercised abilities, one that is independent of determinism; the case is similar to ones discussed below.

[23] This case is a descendant of a family of counterexamples given by Kearns (2011). The cases discussed by Kearns were originally given as counterexamples to an inference rule used in van Inwagen's original presentation of the Direct Argument; the rule in question is in effect the responsibility-theoretic analogue of the rule ALPHA.

[24] The rigidifying interpretation of 'actually' results in $\Box @p$ being true provided $@p$ is true:

RIGIDITY: $\vdash @p \rightarrow \Box @p$

RIGIDITY is by now fairly standard; see Crossley and Humberstone (1977).

has a choice about some necessary truth. So, once again, we have a counterexample to CLOSURE.[25]

Lampert and Merlussi also produce further tempting counterexamples to CLOSURE that do not rely on an 'actually' operator but instead employ rigid definite descriptions and other theoretical tools familiar to contemporary metaphysicians.[26] The authors moreover suggest that having a choice about a proposition can be understood, broadly, with ground-theoretic notions: one has a choice about a proposition when its truth obtains *in virtue of* one's actions, and the truth of a necessary proposition such as *that Nina actually raises her hand* could be said to obtain in virtue of Nina's actions (see Lampert and Merlussi 2021a: 452).[27]

These may serve as putative counterexamples to CLOSURE. And, in line with the anti-incompatibilist insight of the Intensional Argument, they require understanding N in a hyperintensional manner.[28]

Now, we began by showing that any case where one has a choice about some

---

[25]Turner and Capes (2018) respond to related cases due to Kearns (2011). See Lampert and Merlussi (2021b) for a relevant reply.

[26]See Lampert and Merlussi (2021b) as well as Merlussi and Lampert (2023).

[27]Lampert and Merlussi (2021a, 2021b) motivate a two-dimensional account of *rendering a proposition false*, according to which, in a certain sense, one can render so-called *superficially* necessary truths false. Still, on the two-dimensional account INTENSIONALITY and, consequently CLOSURE, both fail.

[28]Hermes (2013) makes the related point that in truthmaker logics, principles such as BETA $\Box$ (the responsibility-theoretic analogue of CLOSURE) from Warfield's Direct Argument are plausibly invalidated. This aligns with the Intensional Argument's implications, since truthmaker logics are typically—if not invariably—hyperintensional (see Restall 1996). We note, in turn, that Schnieder (2004) takes issue with van Inwagen's notion of *rendering a proposition false* and offers his own account of this notion with the sentential and hyperintensional *because* operator (see also Schnieder 2011: 445):

> RENDER: x can render p false (true) $=_{df}$ x can do something such that if x did it, *because of that*, p would be false (true). (2004: 418)

Schnieder is interested in a notion of 'rendering' that applies to both true and false propositions, and explicitly states that he is not concerned with the modal version of the Consequence Argument, but rather with a different version that relies on different principles (see Schnieder 2004: 424, fn. 1). Still, we can define the N operator in the spirit of Schnieder's RENDER:

$Np =_{df} p$ and no one can do something such that if one did it, *because of that* p would be false.

It is not obvious whether N so defined is hyperintensional. If 'because' were to express something like 'just because', then, perhaps, a case for the hyperintensionality of N so defined could be built as follows. Suppose p is some proposition such that Np is true. The disjunction $p \vee \bot$ is necessarily equivalent to p, but one could defend that $N(p \vee \bot)$ is false, because no one can do something such that if one did it, just because of that $p \vee \bot$ would be false—as this would also be false because of $\bot$. De Rizzo (2022) also makes use of the *because* operator to define distinct notions of *choice about a proposition*. For one of these notions, which is plausibly hyperintensional, he claims that CLOSURE fails, formulating cases similar to those suggested by Lampert and Merlussi, Kearns, and Hermes—though the latter two focus on the notion of moral responsibility.

necessary truth is a counterexample to CLOSURE, and the cases so far discussed have all been cases of this sort. Lest the reader conclude that all potential counterexamples worth their salt are like this, we wish to note that similar cases arise concerning contingent *a priori* truths about which no one has a choice. These, in turn, entail contingent truths about which one *does* have a choice, thereby once again violating CLOSURE.

Consider the following case.[29] Take any actual truth p about which someone has a choice. If you like, let p be the truth that Nina raises her hand. Consider now the logical truth $@p \rightarrow p$.[30] Does anyone have a choice about this? Though Nina has a choice about whether or not she raises her hand, plausibly no one, not even Nina, has a choice about whether Nina's raising her hand is materially implied by Nina's *actually* raising her hand; the latter is an apparently trivial, *a priori* truth. But it is also a *contingent* truth: it is true in the actual world and false in any world where p is false.[31] We therefore have two truths, $@p \rightarrow p$ and p; nobody has a choice about the first, whereas someone does have a choice about the second. Since the first entails the second (assuming p is actually true), this is a counterexample to CLOSURE.[32] Since this phenomenon is very general, we therefore have a whole family of further, candidate counterexamples to CLOSURE.

The previous argument invokes a schema from standard modal logic that we call ACT:

$$@p \rightarrow \Box((@p \rightarrow p) \rightarrow p)$$

In the presence of some plausible assumptions, ACT gives us a proper *reductio* of CLOSURE, which by our lights further bolsters the case against CLOSURE. Suppose first that there is *some* actual truth nobody has a choice about; let it be p. Suppose next that nobody has a choice about the logical truth $@p \rightarrow p$. Thus we have $@p$, Np and $N(@p \rightarrow p)$. In the presence of all three, ACT and CLOSURE

[29]See Lampert (2025: 17, fn. 20).

[30]Related to our earlier remark about validity (see footnote 5), here we are not so much iterested in whether this truth is really a logical truth; it suffices for our purposes that it is a truth *and* that it is a theorem, roughly put, of standard modal logic.

[31]It is thus a contingent logical truth, in the sense discussed by Zalta (1988). For some relevant considerations about action and logical truth from a different angle, see Lampert and Waldrop (2023).

[32]To see that the first does indeed entail the second, note that if p is true, so is $@p$, and so given the rigidyfing behavior of the actuality operator we have both $@p$ and $@p \rightarrow \Box @p$, and thus $\Box((@p \rightarrow p) \rightarrow p)$.

generate a contradiction.[33] Since ACT is provable in standard modal logic, one could therefore claim that the blame must fall squarely on CLOSURE.

Now, incompatibilists may well object to $N(@p \to p)$ on the grounds that one could have rendered p, and therefore $@p \to p$, false. After all, if p is actually true, then $@p \to p$ must be false in any circumstance where p is false, and so one could falsify $@p \to p$ in any circumstance where one could falsify p. But notwithstanding these considerations, denying $N(@p \to p)$ remains somewhat puzzling. As we see it, this imposes a certain argumentative burden on anyone committed to the validity of CLOSURE (and so also to N's intensionality), for it shows that CLOSURE is inconsistent with the plausible claim that no one has a choice about whether $@p$ materially implies p, which once again seems plausible *even* where p is a truth someone has (or once had) a choice about.

The issues here are subtle and far-reaching. Earlier, we saw that an important motivation for INTENSIONALITY, and thus for CLOSURE, is the view that propositions are individuated by necessary equivalence, which is entailed by the possible-worlds theory of propositions. But now consider the following argument, showing a strengthening of ACT:

1.	$@p \to \Box @p$	RIGIDITY
2.	$\Box(@p \to ((@p \leftrightarrow p) \leftrightarrow p))$	Necessitated Tautology
3.	$\Box @p \to \Box((@p \leftrightarrow p) \leftrightarrow p)$	2, by K Axiom and Modus Ponens
4.	$@p \to \Box((@p \leftrightarrow p) \leftrightarrow p)$	1 and 3, by Hypothetical Syllogism

Thus, assuming the possible-worlds theory of propositions, given the truth of $@p$ it follows that $@p \leftrightarrow p$ and p are the same proposition. So, for any actually true p, the assumption that no one has a choice about $@p \leftrightarrow p$ leads directly to the claim that no one has a choice about p. In other words, given the general (and, to us, *prima facie* plausible) claim that nobody has a choice about logical truths, the possible-worlds theory of propositions overgenerates, yielding the fatalistic conclusion that nobody has a choice about anything. To avoid this conclusion, incompatibilist advocates of the possible-worlds theory of propositions can be expected, once again, to say that we *do* have a choice about *some* logical truths. On the other hand, those who insist that we do not have a choice about logical truths can be expected

[33] Proof:

1.	$@p$	Premise
2.	$\neg Np$	Premise
3.	$N(@p \to p)$	Premise
4.	$\Box((@p \to p) \to p)$	1 and ACT, by Modus Ponens
5.	Np	3 and 4, by CLOSURE
6.	$\neg Np \wedge Np$	2 and 5, by $\wedge$-Intro.
7.	$\bot$	6, by $\bot$-Intro.

This argument is closely related to a puzzle discussed by Lampert and Waldrop (2023).

to reject the possible-worlds theory of propositions—or some other coarse-grained theory identifying necessarily equivalent propositions.

All of this brings out that the opposing verdicts here—endorsing or rejecting $N(@p \leftrightarrow p)$—cannot simply be opposed *intuitive* verdicts, but they have to interface with a theorist's larger, more global commitments about our abilities *vis-a-vis* modality and logical truth. This general point also applies, *mutatis mutandis*, to the sort of modal semantics for N discussed in the previous section.[34]

Taking stock, it is important to make a final point about the foregoing material. What we have seen is a handful of putative counterexamples to CLOSURE. One might be tempted to discount one or another of these examples on the grounds that they involve rarefied pieces of logical machinery or abstruse phenomena such as the contingent *a priori*. And these cases may simply appear bizzare, which could lead some theorists—with their theory-building leashed to common sense—to ignore them altogether. Both impulses should be resisted. The curious resources these cases appeal to (the contingent *a priori*, actuality operators, ridigified descriptions, etc.) are well-understood; in constructing these cases, nothing is assumed that cannot be motivated by "business as usual" in contemporary semantics and modal metaphysics. Moreover, these counterexamples and the associated problems for CLOSURE are systematic: what they bring out are not simply parochial tensions between CLOSURE and particular cases, but they instead illustrate issues with CLOSURE that are more general.

This, finally, is *exactly* the sort of upshot one could have expected as a takeaway from the Intensional Argument, with its lesson that any counterexamples to CLOSURE must involve hyperintensionality. If N is an intensional operator, the lesson of the Intensional Argument is that CLOSURE is valid, and so are the four arguments discussed in §2. This, we think, really does give those arguments a new lease on life: defenders of such arguments can fruitfully turn their attention to arguing that N is an intensional operator. But there is a corresponding upshot as well for those of us who are skeptical of the arguments: to undercut these arguments, one can investigate systematic pathways to the conclusion that N is a hyperintensional operator. As the foregoing discussion suggests, some recent contributions to the literature have already been pointing in this direction. There are well-understood pathways for motivating hyperintensionality in metaphysics, and what we have seen is that these appear no less fruitful in the metaphysics of free will. Accordingly, it is not difficult to imagine what a rigorous case against CLOSURE might look like.

[34] As before, parallel arguments employing rigidified definite descriptions and other tools are available, as exemplified in Lampert and Merlussi (2021b), Merlussi and Lampert (2023), and, once again, in Lampert and Waldrop (2023).

7. Conclusion

The main arguments challenging free will considered here are valid if CLOSURE is valid. We have shown that, under plausible assumptions, CLOSURE is valid just in case the *N* operator is intensional. This is the core insight of the Intensional Argument: the validity of CLOSURE turns on whether *N* is intensional. We have offered preliminary reasons to think that it is. Drawing on the literature surrounding the Consequence Argument, we identified systematic considerations that support the intensionality of *N*.

However, the Intensional Argument also offers strategic guidance for critics of the arguments discussed in §2 and related incompatibilist theses. For CLOSURE to fail, *N* must be hyperintensional. Thus, promising counterexamples to CLOSURE must track violations of intensionality. This observation helps impose some order on the most compelling counterexamples to CLOSURE in the literature, which all involve hyperintensionality. The cases we surveyed strike us as significant. On their basis, we are tentatively inclined to think that *N* is indeed hyperintensional, and that CLOSURE is invalid. At the same time, the theoretical case for the intensionality of *N* remains strong. The possible-worlds theory of propositions is elegant, powerful, and enjoys considerable theoretical advantages. Similar virtues characterize the standard modal semantics for *N*.

Despite our own inclinations, the broader dialectical upshot is clear. If *N* can be shown to be an intensional operator, the Intensional Argument secures CLOSURE and, with it, the validity of powerful arguments challenging free will. Advocates of these arguments should therefore concentrate their efforts on defending the intensionality of *N*. Conversely, those aiming to challenge the validity of these arguments should focus on considerations that support treating *N* as hyperintensional.

Acknowledgements. For comments and feedback on drafts of the present paper, we are grateful to John Martin Fischer, Pedro Merlussi, Joe Campbell, James Reilly, Kevin Jia, Mike Rea, Kris McDaniel, anonymous referees, and an editor of this journal. For personal conversations on these topics, we are grateful to Dominic LaMantia and Brian Cutter. Warm thanks are also due to the audience at the *Knowledge in Crisis* Seminar in Vienna, as well as to audiences at the 2022 Midwest Regional and 2023 Eastern Regional meetings of the Society of Christian Philosophers. This research was funded in whole or in part by the Austrian Science Fund (FWF) [10.55776/COE3]. For open access purposes, the corresponding author has applied a CC BY public copyright license to any author-accepted manuscript version arising from this submission.

References

Anscombe, G. E. M. (1957). *Intention*. Oxford: Blackwell.

Campbell, Joseph (2007). Free Will and the Necessity of the Past. *Analysis* 67: 105–111.

Campbell, Joseph (2016). The Consequence Argument. In K. Timpe, M. Griffith, and N. Levy (eds.), *The Routledge Companion to Free Will*, Boca Raton, FL: Routledge, an imprint of Taylor and Francis, 151–165.

Campbell, Joseph and Lota, Kenji. (2023). The Consequence Argument and the Mind Argument. In Campbell et al. (2023), 125–143.

Campbell, Joseph, Mickelson, Kristin, and White, V. Alan (2023). *A Companion to Free Will*. Wiley-Blackwell.

Carlson, Erik. (2000). Incompatibilism and the Transfer of Power Necessity. *Nous* 34(2): 277–290.

Crossley, John and Humberstone, Lloyd (1977). The Logic of "Actually". *Reports on Mathematical Logic* 8: 11–29.

De Rizzo, Julio (2022). No Choice For Incompatibilism. *Thought*, 11(1): 6–13.

Dorr, Cian. (2016). Against Counterfactual Miracles. *The Philosophical Review* 125(2): 241–286.

Finch, Alicia and Rea, Michael (2008). Presentism and Ockham's Way Out. *Oxford Studies in Philosophy of Religion* 1: 1–17.

Finch, Alicia and Warfield, Ted (1998). The MIND Argument and Libertarianism. *Mind* 127: 515–528.

Finch, Alicia and Warfield, Ted (1999). Fatalism: Logical and Theological. *Faith and Philosophy* 16(2): 233–238.

Fischer, John Martin (1994) *The Metaphysics of Free Will: An Essay on Control*, Oxford: Blackwell.

Fischer, John Martin (2016). *Our Fate: Essays on God and Free Will*. Oxford: Oxford University Press.

Fischer, John Martin and Todd, Patrick. (2013). The Truth about Foreknowledge. *Faith and Philosophy* 30(3): 286–301.

Fischer, John Martin and Todd, Patrick (eds.) (2015). *Freedom, Fatalism, and Foreknowledge*. Oxford University Press.

Ginet, Carl (1990). On Action. Cambridge: Cambridge University Press.

Haack, Susan (1974). *Deviant Logic, Fuzzy Logic: Beyond the Formalism*. Chicago: University of Chicago Press.

Hermes, Charles (2013). Truthmakers and the Direct Argument. *Philosophical Studies* 167(2): 401–418.

Huemer, Michael. (2000). Van Inwagen's Consequence Argument. *Philosophical Review* 109(4): 525–544.

Kane, Robert (2005). *A Contemporary Introduction to Free Will.* Oxford: Oxford University Press.

Kane, Robert (2007). Libertarianism. In J. M. Fischer, R. Kane, D. Pereboom, and M. Vargas (eds.), *Four Views on Free Will*, Oxford: Blackwell, 5–43.

Kearns, Stephen (2011). Responsibility for Necessities. *Philosophical Studies* 155: 307–24.

Kratzer, Angelika (1977). What "Must" and "Can" Must and Can Mean. *Linguistics and Philosophy* 1(3): 337–355.

Lampert, Fabio (2022). A Puzzle About the Fixity of the Past. *Analysis* 82(3): 426–434.

Lampert, Fabio (2025). Freedom, Omniscience and the Contingent A Priori. *Mind* 134(533): 1–32.

Lampert, Fabio and Merlussi, Pedro (2021a). Counterfactuals, Counteractuals, and Free Choice. *Philosophical Studies* 178: 445–469.

Lampert, Fabio and Merlussi, Pedro (2021b). How (Not) to Construct Worlds With Responsibility. *Synthese* 199: 10389–10413.

Lampert, Fabio and Waldrop, John William (2023). A Puzzle About Moral Responsibility. *Philosophical Studies* 180(8): 2291–2307.

Lewis, David (1981). Are We Free to Break the Laws? *Theoria* 47(3): 113–121.

McKay, Thomas and Johnson, David (1996). A Reconsideration of an Argument Against Compatibilism. *Philosophical Topics* 24: 113–122.

Merlussi, Pedro (2022). Revisiting McKay and Johnson's Counterexample to (β). *Philosophical Explorations* 25(2): 189–203.

Merlussi, Pedro and Lampert, Fabio (2023). Naming and Free Will. *Grazer Philosophische Studien* 99(4): 475–484.

Merricks, Trenton (2009). Truth and Freedom. *Philosophical Review* 118(1): 29–57.

Merricks, Trenton (2011). Foreknowledge and Freedom. *Philosophical Review* 120(4): 567–586.

Mumford, Stephen and Anjum, Rani Lill (2014). A New Argument Against Compatibilism. *Analysis* 74(1): 20–25.

Nelkin, Dana (2001). The Consequence Argument and the Mind Argument. *Analysis*, 61(2): 107–115.

Perry, John (2008). Can't we all just be Compatibilists?: A critical study of John Martin Fischer's My Way. *The Journal of Ethics*, 12: 157–166.

Pike, Nelson (1965). Divine Omniscience and Voluntary Action. *The Philosophical Review*, 74: 27–46.

Pruss, Alexander (2013). Incompatibilism Proved. *Canadian Journal of Philosophy* 43(4): 430–437.

Rea, Michael (2006). Presentism and Fatalism. *Australasian Journal of Philosophy* 84(4): 511–524.

Restall, Greg (1996). Truth-makers, Entailment and Necessity. *Australasian Journal of Philosophy* 74(2): 331–340.

Saunders, John (1966). Of God and freedom. *The Philosophical Review*, 75: 219–225.

Schnieder, Benjamin (2004). Compatibilism and the Notion of Rendering Something False. *Philosophical Studies* 117(3): 409–428.

Schnieder, Benjamin (2011). A Logic for 'Because'. *Review of Symbolic Logic* 4(3): 445–465.

Spencer, Jack (2017). Able to do the Impossible. *Mind*, 126(502): 466–497.

Taylor, Richard (1962). Fatalism. *The Philosophical Review*, 71: 56–66.

Todd, Patrick (2023). Foreknowledge Requires Determinism. *Philosophy and Phenomenological Research* 107(1): 125–146.

Turner, P. Roger and Capes, Justin (2018). Rule A. *Pacific Philosophical Quarterly* 99: 580–595.

Van Inwagen, Peter (1983). *An Essay on Free Will*. Oxford: Oxford University Press.

Van Inwagen, Peter (1989). When is the Will Free? *Philosophical Perspectives* 3: 399–422.

Van Inwagen, Peter (1990). Logic and the Free Will Problem. *Social Theory and Practice* 16: 279–290.

Van Inwagen, Peter (2000). Free Will Remains a Mystery. *Philosophical Perspectives* 14: 1–19.

Van Inwagen, Peter (2014). *Metaphysics*. New York: Routledge.

Van Inwagen, Peter (2015). Some Thoughts on An Essay on Free Will. *The Harvard Review of Philosophy* 22: 16–30.

Van Inwagen, Peter (2017). Thinking About Free Will. *Cambridge: Cambridge University Press*.

Vihvelin, Kadri (1988). The Modal Argument for Incompatibilism. *Philo-*

sophical Studies 53(2): 227–244.

Waldrop, John William (2023). A Problem for Counterfactual Sufficiency. *Analysis* 83(3): 527–535.

Warfield, Ted (1996). Determinism and Moral Responsibility Are Incompatible. *Philosophical Topics* 24(2): 215–226.

Widerker, David (1987). On an Argument for Incompatibilism. *Analysis* 47: 37–41.

Williamson, Timothy and Stanley, Jason (2001). Knowing How. *The Journal of Philosophy* 98(8): 411–444.

Zalta, Edward (1988). Logical and Analytic Truths That Are Not Necessary. *Journal of Philosophy* 85(2): 57–74.